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Influence of viscous dissipation and radiation on unsteady MHD free-convection flow past an infinite heated vertical plate in a porous medium with time-dependent suction

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Abstract

This paper investigates the influence of viscous dissipation and radiation on the problem of unsteady magnetohydrodynamic free-convection flow past an infinite vertical heated plate in an optically thin environment with time-dependent suction. By taking the radiative heat flux in the differential form, and imposing an oscillatory time-dependent perturbation, the coupled nonlinear problem is solved for the temperature and velocity distributions. The effects of the material parameters on the temperature and velocity profiles are discussed quantitatively. The results show that increased cooling ($Gr > 0$) of the plate and the Eckert number leads to a rise in the velocity profile; while increases in magnetic field, radiation and Darcy parameters are associated with decrease in the velocity. Also, an increase in the Eckert number leads to an increase in the temperature, whereas increases in radiation and magnetic field parameters lead to a decrease in the temperature distribution when the plate is being cooled.

1. Introduction

Flow through porous medium past infinite vertical plate is common in nature and has many applications in engineering and science. A number of workers have investigated such flows and excellent literature on the properties and phenomenon may be found in literature [1–5]. For example, Soundalgekar [4] investigated the effects of free-convection currents on the oscillatory type boundary layer flow past an infinite vertical plate with constant suction where the plate temperature differs from the free stream temperature.

Recently, attention has been on the effects of transversely applied magnetic field and thermal perturbation on the flow of electrically conducting viscous fluids such as plasma. Various properties associated with the interplay of magnetic fields and thermal perturbation in porous medium past vertical plate find useful applications in astrophysics, geophysical fluid dynamics, and engineering. Researches in these fields have been conducted by many investigators [6–13]. For example, Soundalgekar [6] investigated a two dimensional steady free-convection flow of an incompressible, viscous, electrically conducting fluid past an infinite vertical porous plate with constant suction and plate temperature when the difference between the plate temperature and free stream is moderately large to cause free-convection currents. In another study Israel-Cookey and Sigalo [13] investigated the problem of unsteady MHD past a semi-infinite vertical plate in an optically thin environment with simultaneous effects of radiation, free-convection parameters and time-dependent suction.

Most of the previous studies of the same problem neglected viscous dissipation. This present study is an attempt to complement the earlier works of Soundalgekar [6] and Israel-Cookey and Sigalo [13] by investigating the simultaneous effects of viscous dissipation

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Nomenclature

A	small positive parameter
T'_w	wall temperature
T_∞	reference temperature
U'	dimensional free stream velocity
(u', v', w')	dimensional velocity components
(x', z')	dimensional Cartesian coordinates
t'	dimensional time
g	acceleration due to gravity
w'_0	dimensional suction velocity
$H_0'^2$	constant transverse magnetic field
k	dimensional porosity parameter
c_p	specific heat capacity
M^2	nondimensional magnetic parameter
Pr	Prandtl number
Gr	Grashof number (free-convection parameter)
R^2	radiation parameter
U_0	mean velocity of $U'(t')$
q'_z	radiative heat flux
B	Planck's function
Ec	Eckert number

Greek symbols

ε	small positive parameter
β	Stephan–Boltzman constant
$\nu = \mu/\rho$	kinematic viscosity
σ_c	electrical conductivity
μ	permeability
ρ	fluid density
ω'	dimensional free stream frequency of oscillation
κ	thermal conductivity
α^2	absorption coefficient
χ^2	Darcy number
δ	radiation absorption coefficient
λ	frequency

and radiation to the problem of unsteady MHD flow past an infinite vertical heated plate in a porous medium with time-dependent suction. This attempt therefore, widens the applicability of problems of this nature.

2. Mathematical formulation

We consider the unsteady flow of an incompressible viscous, radiating hydromagnetic fluid past an infinite porous heated vertical plate with time-dependent suction in an optically thin environment. The physical model and the coordinate system are shown in Fig. 1. The x'-axis is taken along the vertical infinite porous plate in the upward direction and the z'-axis normal to the plate.

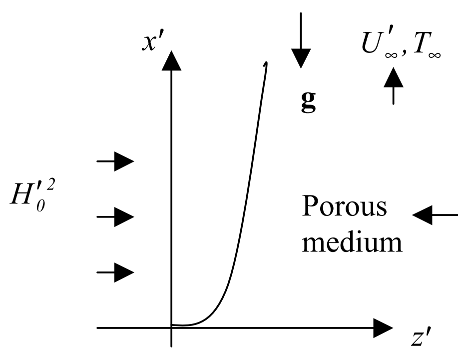

Fig. 1. The physical model and coordinate system of the problem.

At time $t' = 0$, the plate is maintained at a temperature T'_w, which is high enough to initiate radiative heat transfer. A constant magnetic field $H_0'^2$ is maintained in the z' direction and the plate moves uniformly along the positive x' direction with velocity U_0. Under Boussinesq approximation the flow is governed by the following equations:

$$\frac{\partial w'}{\partial z'} = 0 \tag{1}$$

$$\frac{\partial u'}{\partial t'} + w'\frac{\partial u'}{\partial z'} = \nu\frac{\partial^2 u'}{\partial z'^2} + \frac{\partial U'}{\partial t'} - \left(\frac{\mu^2\sigma_c H_0'^2}{\rho} + \frac{\nu}{k}\right)(u' - U') + g\beta(T' - T_\infty) \tag{2}$$

$$\frac{\partial T'}{\partial t'} + w'\frac{\partial T'}{\partial z'} = \frac{\kappa}{\rho c_p}\left(\frac{\partial^2 T'}{\partial z'^2} - \nabla q'_{z'}\right) + \frac{\mu}{\rho c_p}\left(\frac{\partial u'}{\partial z'}\right)^2 \tag{3}$$

$$\frac{\partial^2 q'_{z'}}{\partial z'^2} - 3\alpha^2 q'_{z'} - 16\alpha\sigma T_\infty^3\frac{\partial T'}{\partial z'} = 0 \tag{4}$$

The boundary conditions are

$$u' = 0, \quad T' = T'_w \quad \text{on } z' = 0$$
$$u' = U'(t') = w'_0(1 + \varepsilon e^{i\omega' t'}), \quad T' = T_\infty \quad \text{as } z' \to \infty \tag{5a, b}$$

Since the medium is optically thin with relatively low density and $\alpha \ll 1$ the radiative heat flux given by Eq. (4) in the spirit of Cogley et al. [14] becomes

$$\frac{\partial q'_{z'}}{\partial z'} = 4\alpha^2(T' - T_\infty) \tag{6a}$$

where

$$\alpha^2 = \int_0^\infty \delta\lambda \frac{\partial B}{\partial T'} \tag{6b}$$

Further, from Eq. (1) it is clear that w' is a constant or a function of time only and so we assume

$$w' = -w_0\left(1 + \varepsilon A\, \mathrm{e}^{\mathrm{i}\omega' t'}\right) \tag{7}$$

such that $\varepsilon A \ll 1$, and the negative sign indicates that the suction velocity is towards the plate.

In view of Eqs. (4), (6) and (7), Eqs. (2) and (3) become

$$\frac{1}{4}\frac{\partial u}{\partial t} - \left(1 + \varepsilon A\, \mathrm{e}^{\mathrm{i}\omega t}\right)\frac{\partial u}{\partial z} = \frac{1}{4}\frac{\partial U}{\partial t} + \frac{\partial^2 u}{\partial z^2} - (M^2\chi^2)(u - U) + Gr\theta \tag{8}$$

$$\frac{1}{4}Pr\frac{\partial \theta}{\partial t} - Pr\left(1 + \varepsilon A\, \mathrm{e}^{\mathrm{i}\omega t}\right)\frac{\partial \theta}{\partial z} = \left(\frac{\partial^2}{\partial z^2} - R^2\right)\theta + PrEc\left(\frac{\partial u}{\partial z}\right)^2 \tag{9}$$

where we have used the following dimensionless variables

$$\begin{aligned}
&z = \frac{w_0'}{\nu}z', \quad t = \frac{w_0'^2}{4\nu}t', \quad u = \frac{u'}{U_0}, \quad \omega = \frac{4\nu}{w_0'^2}\omega', \\
&U = \frac{U'}{U_0}, \quad \theta = \frac{T' - T_\infty}{T_\mathrm{w} - T_\infty}, \quad \chi^2 = \frac{\nu^2}{k w_0'^2}, \quad Pr = \frac{\mu c_p}{\kappa}, \\
&Gr = \frac{\nu g\beta(T_\mathrm{w} - T_\infty)}{U_0 w_0'^2}, \quad Ec = \frac{U_0^2}{c_p(T_\mathrm{w} - T_\infty)}, \\
&R^2 = \frac{4\alpha^2}{\rho c_p \kappa w_0'^2}(T_\mathrm{w} - T_\infty), \quad M^2 = \frac{\mu^2\sigma_\mathrm{e} H_0'^2}{\rho w_0'^2}
\end{aligned} \tag{10}$$

Eqs. (8) and (9) are now subject to the boundary conditions

$$\begin{aligned}
&u = 0, \quad \theta = 1 \quad \text{on } z = 0 \\
&u \to 1 + \varepsilon\mathrm{e}^{\mathrm{i}\omega t}, \quad \theta \to 0 \quad \text{as } z \to \infty
\end{aligned} \tag{11}$$

The mathematical statement of the problem is now complete and embodies the solution of Eqs. (8) and (9) subject to boundary conditions (11).

3. Method of solution

The problem posed in Eqs. (8) and (9) subject to the boundary conditions presented in Eq. (11) are highly nonlinear equations and generally will involve a step by step numerical integration of the explicit finite difference scheme. However, analytical solutions could be possible. Since ε is small we can advance by adopting regular perturbation expansion of the form

$$u(z,t) = u_0(z) + \varepsilon u_1(z)\,\mathrm{e}^{\mathrm{i}\omega t} \tag{12a}$$

$$\theta(z,t) = \theta_0(z) + \varepsilon \theta_1(z)\,\mathrm{e}^{\mathrm{i}\omega t} \tag{12b}$$

Substituting Eq. (12) in Eqs. (8), (9), (11), neglecting the coefficients of $\mathrm{O}(\varepsilon^2)$ and simplifying we obtain the sequence of approximations

$$u_0'' + u_0' - (\chi^2 + M^2)u_0 = -(\chi^2 + M^2) - Gr\theta_0 \tag{13}$$

$$\theta_0'' + Pr\theta_0' - R^2\theta_0 = -PrEc\left(\frac{\partial u_0}{\partial z}\right) \tag{14}$$

subject to the boundary conditions

$$\left.\begin{aligned}
&u_0 = 0, \quad \theta_0 = 1 \quad \text{on } z = 0 \\
&u_0 \to 1, \quad \theta_0 \to 0 \quad \text{as } z \to \infty
\end{aligned}\right\} \tag{15}$$

for O(1) equations, and

$$u_1'' + u_1' - \left(\chi^2 + M^2 + \frac{\mathrm{i}\omega}{4}\right)u_1 = -Au_0' - Gr\theta_1 - \left(\chi^2 + M^2 + \frac{\mathrm{i}\omega}{4}\right) \tag{16}$$

$$\theta_1'' + Pr\theta_1' - \left(R^2 + \frac{\mathrm{i}\omega}{4}\right)\theta_1 = -PrA\theta_0' - 2PrEcu_0'u_1' \tag{17}$$

subject to

$$\left.\begin{aligned}
&u_1 = 0, \quad \theta_1 = 0 \quad \text{on } z = 0 \\
&u_1 \to 1, \quad \theta_1 \to 0 \quad \text{as } z \to \infty
\end{aligned}\right\} \tag{18}$$

for $\mathrm{O}(\varepsilon)$ equations.

To solve the nonlinear-coupled Eqs. (13)–(18), we further assume that the viscous dissipation parameter (Eckert number Ec) is small, and therefore, advance an asymptotic expansion for the flow temperature and velocity as follows:

$$\begin{aligned}
&u_0(z) = u_{01}(z) + Ecu_{02}(z) \\
&\theta_0(z) = \theta_{01}(z) + Ec\theta_{02}(z) \\
&u_1(z) = u_{11}(z) + Ecu_{12}(z) \\
&\theta_1(z) = \theta_{11}(z) + Ec\theta_{12}(z)
\end{aligned} \tag{19}$$

Substituting Eq. (19) into Eqs. (13)–(18), we obtain the following sequence of approximations:

$$u_{01}'' + u_{01}' - (\chi^2 + M^2)u_{01} = -Gr\theta_{01} - (\chi^2 + M^2) \tag{20}$$

$$\theta_{01}'' + Pr\theta_{01}' - R^2\theta_{01} = 0 \tag{21}$$

$$u_{02}'' + u_{02}' - (\chi^2 + M^2)u_{02} = -Gr\theta_{02} \tag{22}$$

$$\theta_{02}'' + Pr\theta_{02}' - R^2\theta_{02} = -Pru_{01}'^2 \tag{23}$$

subject to

$$\left.\begin{aligned}
&u_{01} = 0 = u_{02}, \quad \theta_{01} = 1, \quad \theta_{02} = 0 \quad \text{on } z = 0 \\
&u_{01} = 1, \quad u_{02} = 0, \quad \theta_{01} = 0 = \theta_{02} \quad \text{as } z \to \infty
\end{aligned}\right\} \tag{24}$$

for O(1) equations, and

$$u''_{11} + u'_{11} - N_1 u_{11} = -Au'_{01} - N_1 - Gr\theta_{11} \tag{25}$$

$$\theta''_{11} + Pr\theta'_{11} - N_2\theta_{11} = -Pr\theta'_{01} \tag{26}$$

$$u''_{12} + u'_{12} - N_1 u_{12} = -Gr\theta_{12} - Au'_{02} \tag{27}$$

$$\theta''_{12} + Pr\theta'_{12} - N_2\theta_{12} = -2Pru'_{01}u'_{11} - PrA\theta'_{12} \tag{28}$$

subject to

$$\begin{aligned} &u_{11} = u_{12} = \theta_{11} = \theta_{12} = 0 \quad \text{on } z = 0 \\ &u_{11} = 1, \quad u_{12} = \theta_{11} = \theta_{12} = 0 \quad \text{as } z \to \infty \end{aligned} \tag{29}$$

for $O(Ec)$ equations and where $N_1 = \chi^2 + M^2 + i\omega/4$; $N_2 = R^2 + i\omega Pr/4$.

Solving Eqs. (20)–(23) with boundary conditions (24) and Eqs. (25)–(28) satisfying the boundary conditions (29) and substituting into Eq. (19) and using Eq. (12) we obtain the temperature and velocity profiles of the flow respectively as:

$$\begin{aligned} \theta(z) = {} & e^{-m_1 z} + Ec\big(\alpha_1 e^{-2m_2 z} + \alpha_2 e^{-2m_1 z} \\ & + \alpha_3 e^{-(m_1+m_2)z} - (\alpha_1 + \alpha_2 + \alpha_3) e^{-m_3 z}\big) \\ & + \varepsilon e^{i\omega t}\big(\alpha_9(e^{-m_1 z} - e^{-m_5 z}) + Ec\big(A_7 e^{-m_7 z} \\ & + \alpha_{13} e^{-(m_1+m_2)z} + \alpha_{14} e^{-(m_1+m_5)z} + \alpha_{15} e^{-(m_1+m_6)z} \\ & + \alpha_{16} e^{-(m_2+m_5)z} + \alpha_{17} e^{-(m_2+m_6)z} \\ & + \alpha_{18} e^{-2m_1 z} + \alpha_{19} e^{-2m_2 z} + \alpha_{20} e^{-m_3 z}\big)\big) \end{aligned} \tag{30}$$

$$\begin{aligned} u(z) = {} & 1 + A_1 e^{-m_2 z} + A_2 e^{-m_1 z} + Ec\big(A_4 e^{-m_4 z} \\ & + \alpha_4 e^{-m_1 z} + \alpha_5 e^{-2m_2 z} + \alpha_6 e^{-2m_1 z} \\ & + \alpha_7 e^{-(m_1+m_2)z} + \alpha_8 e^{-m_3 z}\big) \\ & + \varepsilon e^{i\omega t}\big(1 - (1 + \alpha_{10} + \alpha_{11} + \alpha_{12}) e^{-m_6 z} + \alpha_{10} e^{-m_2 z} \\ & + \alpha_{11} e^{-m_1 z} + \alpha_{12} e^{-m_5 z} + Ec\big(A_8 e^{-m_8 z} + \beta_1 e^{-m_7 z} \\ & + \beta_2 e^{-(m_1+m_2)z} + \beta_3 e^{-2m_1 z} + \beta_4 e^{-2m_2 z} \\ & + \beta_5 e^{-m_3 z} + \beta_6 e^{-m_1 z} + \beta_7 e^{-m_4 z} + \beta_8 e^{-(m_1+m_5)z} \\ & + \beta_9 e^{-(m_1+m_6)z} + \beta_{10} e^{-(m_2+m_5)z} + \beta_{11} e^{-(m_2+m_6)z}\big)\big) \end{aligned} \tag{31}$$

where the constants are given in Appendix A.

4. Results and discussion

In the previous sections, we have formulated and solved the problem of the influence of viscous dissipation and radiation with time-dependent suction velocity on the unsteady MHD free-convection flow along a stationary infinite vertical plate in a porous medium. By invoking the optically thin differential approximation for the radiative heat flux in the energy equation, and oscillatory time-dependent perturbation for the flow temperature and velocity. In the numerical computation, the Prandtl number, $Pr = 0.71$ which corresponds to air and various values of the material parameters are used. In addition, the boundary condition $y \to \infty$ is approximated by $y_{max} = 6$, which is sufficiently large for the velocity to approach the relevant stream velocity.

In the subsequent analysis, we started with the temperature profiles due to its primary importance in astrophysical environments. The temperature profiles are presented when the free-convection currents are cooling the plate (see Figs. 2 and 3). From Fig. 2, it is observed that the temperature within the plasma decayed rapidly away from the plate. Also, a greater heating by viscous dissipation caused a rise in temperature. But an increase in radiation and magnetic field parameter recorded a decrease in temperature. From Fig. 3, an increase in the free stream frequency, ω is associated with a decrease in the temperature distribution. In the absence of viscous dissipation these results agrees quantitatively with the earlier results of Israel-Cookey and Sigalo [13].

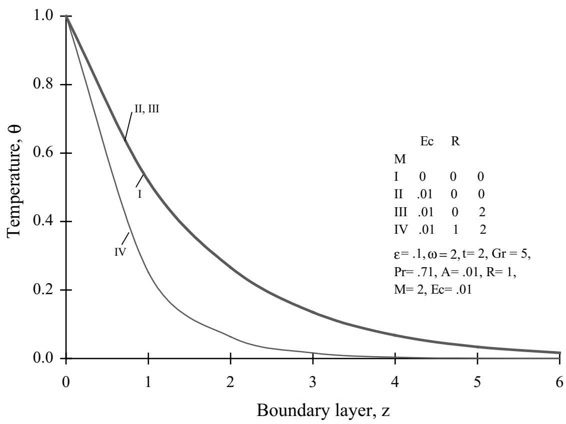

Fig. 2. Temperature profiles against the boundary layer, z for different material parameters.

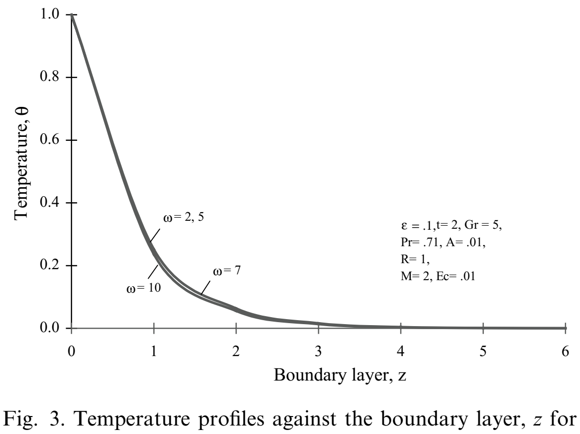

Fig. 3. Temperature profiles against the boundary layer, z for different material parameters.

In Figs. 4–8, we presented the behaviour of the velocity of the flow for various material parameters; (Ec, ω, Gr, M, χ, R, and A) for $Pr = 0.71$. It is observed that the velocity rose steadily and then converged close to the free stream velocity. In the absence of viscous dissipation parameter, Ec, the results for cooling of the plate ($Gr > 0$) agrees quantitatively with earlier results of Israel-Cookey and Sigalo [13]. Equally, for $R = 0$ and $A = 0$, these results are quantitatively consistent with the results of Soundalgekar [6].

We observe from Fig. 4, that an increase in the viscous dissipation parameter, Ec, is associated with an increase in velocity. On cooling the plate with the aid of convection currents (see Figs. 5–7) we observe that separate increase in the material parameters M, R and χ are associated with decreases in the velocity profiles. Furthermore, an increased cooling through convection currents resulted in the rise of the velocity distribution

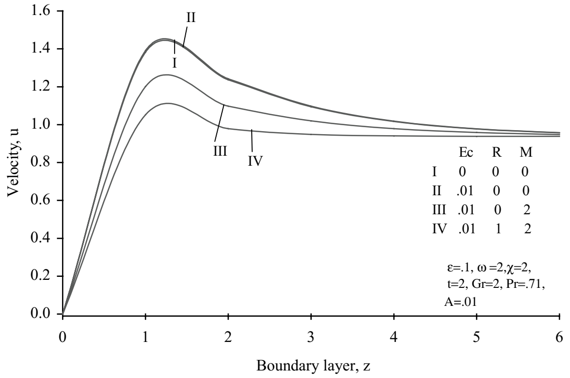

Fig. 4. Velocity profiles against the boundary layer, z for different material parameters.

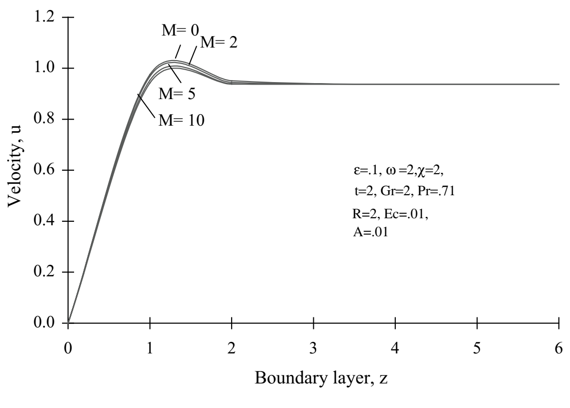

Fig. 5. Velocity profiles against the boundary layer, z for different values of material parameters.

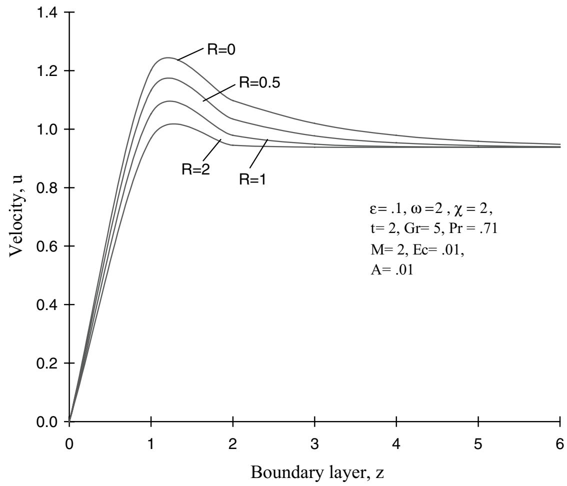

Fig. 6. Velocity profiles against the boundary layer, z for different values of radiation parameter, R.

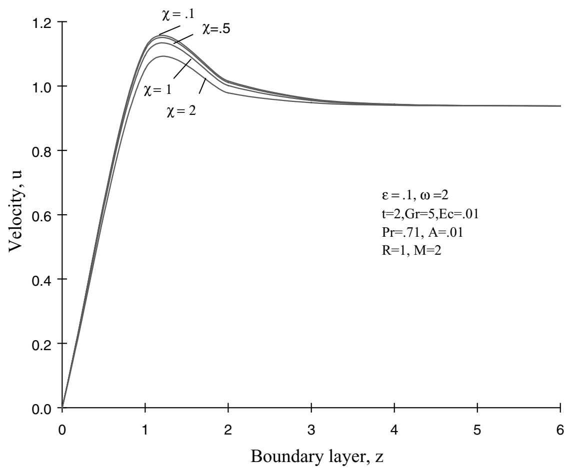

Fig. 7. Velocity profiles against the boundary layer, z for different values of Darcy parameter, χ.

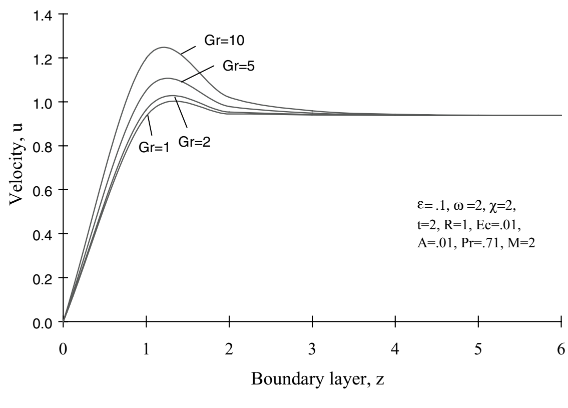

Fig. 8. Velocity profiles against the boundary layer, z for different values of free convection parameter, Gr.

(see Fig. 8). Again, this result is in good quantitative agreement with the results of Soundalgekar [6].

5. Conclusions

In conclusion therefore, the flow of an unsteady MHD free-convection past an infinite vertical plate with time-dependent suction under the simultaneous effects of viscous dissipation and radiation is affected by the material parameters. In addition, an increase temperature profile is a function of an increase in viscous dissipation. Whereas an increase in radiation and magnetic field parameters led to a decrease in the temperature profile on cooling. Equally, cooling of the plate by convection currents with increases in the radiation, magnetic field and Darcy parameters led to a decrease in the velocity profile. Finally, increased cooling of the plate and viscous dissipation resulted in an increase in the velocity profile.

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Appendix A

The followings are the constants that appear in Eqs. (30) and (31).

$$m_1 = m_3 = \frac{1}{2}\left(Pr + \sqrt{Pr^2 + 4R^2}\right)$$
$$m_2 = m_4 = \frac{1}{2}\left(1 + \sqrt{1 + 4(\chi^2 + M^2)}\right)$$
$$m_5 = m_7 = \frac{1}{2}\left(Pr + \sqrt{Pr^2 + 4N_2}\right)$$
$$m_6 = m_8 = \frac{1}{2}\left(1 + \sqrt{1 + 4N_1}\right)$$
$$A_1 = \frac{Gr}{m_1^2 - m_1 - \chi^2 - M^2} - 1$$
$$A_2 = \alpha_4 = -\frac{Gr}{m_1^2 - m_1 - \chi^2 - M^2}$$
$$A_4 = -(\alpha_4 + \alpha_5 + \alpha_6 + \alpha_7 + \alpha_8)$$
$$A_7 = -\sum_{i=13}^{20} \alpha_i$$
$$A_8 = -\sum_{i=1}^{11} \beta_i$$
$$\alpha_1 = -\frac{Prm_2^2A_1^2}{4m_2^2 - 2m_2 - R^2}$$
$$\alpha_2 = -\frac{Prm_1^2A_2^2}{4m_1^2 - 2m_1 - R^2}$$
$$\alpha_3 = -\frac{2Prm_1m_2A_1A_2}{(m_1 + m_2)^2 - (m_1 + m_2) - R^2}$$
$$\alpha_5 = -\frac{Gr\alpha_1}{4m_1^2 - 2m_2^2 - \chi^2 - M^2}$$
$$\alpha_6 = -\frac{Gr\alpha_2}{4m_1^2 - 2m_1 - \chi^2 - M^2}$$
$$\alpha_7 = -\frac{Gr\alpha_3}{(m_1 + m_2)^2 - (m_1 + m_2) - \chi^2 - M^2}$$
$$\alpha_8 = \frac{Gr(\alpha_1 + \alpha_2 + \alpha_3)}{m_3^2 - m_3 - \chi^2 - M^2}$$
$$\alpha_9 = \frac{m_1Pr}{m_1^2 - Prm_1 - N_2}$$
$$\alpha_{10} = \frac{AA_1m_2}{m_2^2 - m_2 - N_1}$$
$$\alpha_{11} = \frac{AA_2m_1 - Gr\alpha_9}{m_1^2 - m_1 - N_1}$$
$$\alpha_{12} = \frac{Gr\alpha_9}{m_5^2 - m_5 - N_1}$$
$$\alpha_{13} = \frac{d_{00}}{(m_1 + m_2)^2 - Pr(m_1 + m_2) - N_2}$$
$$\alpha_{14} = \frac{d_{11}}{(m_2 + m_5)^2 - Pr(m_2 + m_5) - N_2}$$
$$\alpha_{15} = \frac{d_{22}}{(m_1 + m_6)^6 - Pr(m_1 + m_6) - N_2}$$
$$\alpha_{16} = \frac{d_{33}}{(m_2 + m_5)^2 - Pr(m_2 + m_5) - N_2}$$
$$\alpha_{17} = \frac{d_{44}}{(m_2 + m_6)^2 - Pr(m_2 + m_6) - N_2}$$
$$\alpha_{18} = \frac{d_{55}}{4m_1^2 - 2m_1Pr - N_2}$$
$$\alpha_{19} = \frac{d_{66}}{4m_2^2 - 2m_2 - N_2}$$
$$\alpha_{20} = \frac{d_{77}}{m_3^2 - Prm_3 - N_2}$$
$$d_{77} = APrm_3$$
$$d_{00} = APr(m_1 + m_2)\alpha_3 - 2Prm_1m_2A_1\alpha_{11} - 2Prm_1m_2 - 2Prm_1m_2\alpha_{10}$$
$$d_{11} = -2Prm_1m_5\alpha_{12}$$
$$d_{22} = -2Prm_1m_6A_6$$
$$d_{33} = -2Prm_2m_5\alpha_{12}A$$
$$d_{44} = 2Prm_2m_6A_1$$
$$d_{55} = 2APrm_1\alpha_2 - 2Prm_1^2\alpha_{11}$$
$$d_{66} = 2APrm_2\alpha_1 - 2Prm_2^2\alpha_{10}A_1$$

$$\beta_1 = -\frac{GrA_7}{m_7^2 - m_7 - N_1}$$
$$\beta_2 = \frac{A\alpha_7(m_1 + m_2) - Gr\alpha_{13}}{(m_1 + m_2)^2 - (m_1 + m_2) - N_1}$$
$$\beta_3 = \frac{2A\alpha_6 m_1 - Gr\alpha_{18}}{4m_1^2 - 2m_1 - N_1}$$
$$\beta_4 = \frac{2A\alpha_5 m_2 - Gr\alpha_{19}}{4m_2^2 - 2m_2 - N_1}$$
$$\beta_5 = \frac{A\alpha_8 + \alpha_{20}}{m_3^2 - m_3 - N_1}$$
$$\beta_6 = \frac{A\alpha_4 m_1}{m_1^2 - m_1 - N_1}$$
$$\beta_7 = \frac{AA_4 m_4}{m_4^2 - m_4 - N_1}$$
$$\beta_8 = -\frac{Gr\alpha_{14}}{(m_1 + m_5)^2 - (m_1 + m_5) - N_1}$$
$$\beta_9 = -\frac{Gr\alpha_{15}}{(m_1 + m_6)^2 - (m_1 + m_6) - N_1}$$
$$\beta_{10} = -\frac{Gr\alpha_{16}}{(m_2 + m_5)^2 - (m_2 + m_5) - N_1}$$
$$\beta_{11} = -\frac{Gr\alpha_{17}}{(m_2 + m_6)^2 - (m_2 + m_6) - N_1}.$$

References

[1] M.J. Lighthill, The response of laminar skin friction and heat transfer fluctuations in the stream velocity, Proc. Royal Soc. London A 224 (1954) 1–23.

[2] T.J. Stuart, A solution of the Navier–Stokes and energy illustrating the response of skin—friction and temperature of an infinite plate thermometer to fluctuations in the stream velocity, Proc. Royal Soc. London A 231 (1955) 116–130.

[3] V.M. Soundalgekar, Free-convection effects on the oscillatory flow past an infinite vertical porous plate with constant suction I, Proc. Royal Soc. London A 333 (1973) 25–36.

[4] V.M. Soundalgekar, Free-convection effects on the oscillatory flow past an infinite vertical porous plate with constant suction II, Proc. Royal Soc. London A 333 (1973) 37–50.

[5] I. Pop, V.M. Soundalgekar, On the unsteady boundary layers in a rotating flow, J. Inst. Maths Applic. 15 (1975) 343–349.

[6] V.M. Soundalgekar, Free-convection effects on steady MHD flow past a vertical porous plate, J. Fluid Mech. 66 (1974) 541–551.

[7] A.R. Bestman, Transient effect on laminar convection to a radiating gas past a vertical flat plate, AMSE Modell., Simul. Control B 3 (2) (1985) 1–10.

[8] A.K. Singh, C.K. Dikshif, Hydromagnetic flow past a continuously moving semi-infinite plate at large suction, Astrophys. Space Sci. 248 (1988) 249–256.

[9] A.R. Bestman, S.K. Adjepong, Unsteady hydromagnetic free-convection flow with radiative heat transfer in a rotating fluid, Astrophys. Space Sci. 143 (1988) 73–80.

[10] H.S. Takhar, R.S.R. Gorla, V.M. Soundalgekar, Radiative effects on MHD free-convection flow past a semi-infinite vertical plate, Int. J. Numer. Heat Fluid Flow 6 (2) (1996) 77–83.

[11] K.D. Alagoa, G. Tay, T.M. Abbey, Radiative and free-convection effects of a MHD flow through porous medium between infinite parallel plates with time-dependent suction, Astrophys. Space Sci. 260 (1999) 455–468.

[12] Y.J. Kim, Unsteady MHD convective heat transfer past a semi-infinite vertical porous moving plate with variable suction, Int. J. Engng. Sci. 38 (2000) 833–845.

[13] C. Israel-Cookey, F.B. Sigalo, On the unsteady MHD free-convection flow past semi-infinite heated porous vertical plate with time-dependent suction and radiative heat transfer, AMSE Modell., Meas. Control, in press.

[14] A.C.L. Cogley, W.G. Vincenti, E.S. Gilles, Differential approximation for radiative heat transfer in a nonlinear equations-grey gas near equilibrium, Am. Inst. Aeronaut. Astronaut. J. 6 (1968) 551–553.